Mean Trajectory of a Rifle Bullet

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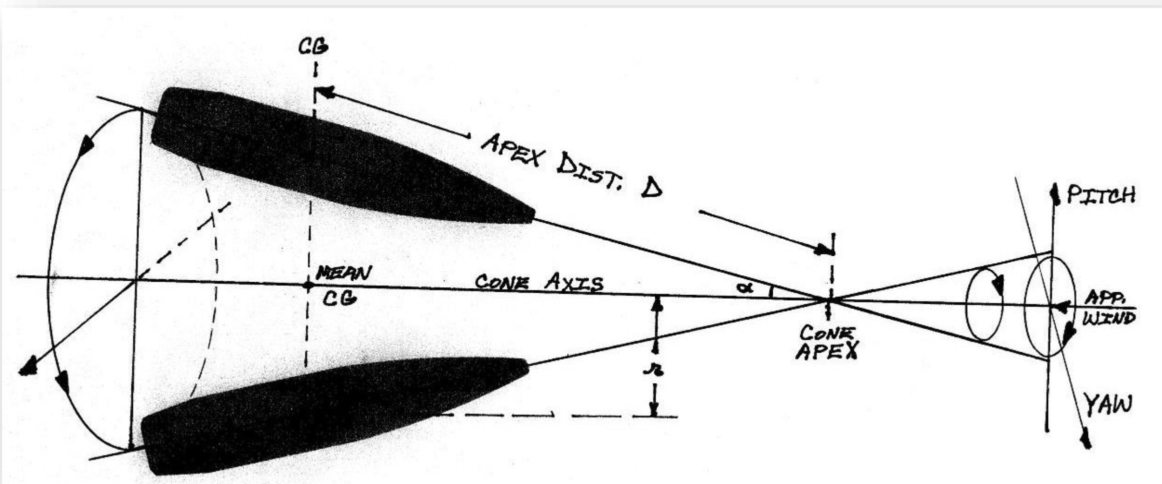
Introduction

The *mean trajectory* of a spin-stabilized rifle bullet in free flight can be defined as the path in 3-space which is followed by the coning center about which the center of gravity (CG) of the coning bullet spirals throughout its flight. We term this center point of the circular coning motion the "mean CG" of the coning bullet. Thus, this ***mean* CG** moves smoothly along the ***mean trajectory*** at the bullet's ***mean forward velocity***. The mean velocity vector **V(t)** is tangent to the mean trajectory at the coning bullet's instantaneous mean CG location.



Coning Rifle Bullet at Extreme Top and Bottom Positions

A non-zero coning angle **α(t)** at any time **t** causes the CG of the coning bullet to follow a helical path of small radius **r(t)** about this mean trajectory:

$$r(t) = D(t)*SIN[\alpha(t)] \qquad (1)$$

where **D(t)** is the gradually lengthening coning distance of the bullet's CG from its coning apex. The coning apex distance **D(t)** is typically about **10 caliber** at long supersonic ranges. The coning radius **r(t)** seldom exceeds about **1.0 calibers** for dynamically stable rifle bullets in a typical flights downrange.

The coning angle **α(t)** is itself a free variable in Coning Theory, along with its associated coning radius **r(t)**. The coning angle **α(t)** *increases* with each change in flight conditions encountered by the bullet in flight and decreases ***only*** with slow-mode damping. Thus, at any time **t** after launch, the coning angle **α(t)** depends upon the entire prior history of the trajectory. The coning half-angle **α(t)** is essentially also the aerodynamic "angle of attack" of the free-flying rifle bullet.

From Coning Theory, the coning distance **D(t)** can be found in feet as:

$$D(t) = q*S*(CL_{\alpha} + CD_{\alpha})/(m*\omega_2^2) \qquad (2)$$

where only the bullet's cross-sectional area **S** in square feet and mass **m** in slugs are invariant over time **t** and airspeed **V(t)** during the flight. The dimensionless coefficients of lift and drag are as they are usually defined in linear aeroballistic theory.

The magnitude of the circular frequency ω_2, the bullet's coning rate in radians per second, can be evaluated from Coning Theory as:

$$\omega_2(t) = q*S*d*CM_{\alpha}/L \qquad (\alpha, L \neq 0) \qquad (3)$$

where the instantaneous magnitude of the bullet's angular momentum **L** equals **Ix*ω(t)**. The dimensionless overturning moment coefficient CM_{α} is also as it is usually defined in linear aeroballistics.

This coning rate $\omega_2(t)$ of the CG of the bullet is *identically equal* to the slow-mode gyroscopic precession rate of its spin-axis of the bullet, also $\omega_2(t)$ radians per second. We usually find the gyroscopic precession rate $\omega_2(t)$ from the Tri-Cyclic theory relationships:

$$\omega_1(t) + \omega_2(t) = (Ix/Iy)*\omega(t)$$
$$R(t) = \omega_1(t)/\omega_2(t) = 2*\{Sg + SQRT[Sg*(Sg - 1)]\} - 1$$
$$\omega_2(t) = [\omega_1(t) + \omega_2(t)]/[R(t) + 1] \qquad (4)$$

with **Sg≥1.5** being the bullet's gyroscopic stability factor at any point during its flight. The instantaneous spin-rate of the bullet is **ω(t)** in radians per second. **Ix** is the second moment of inertia of the bullet's mass distribution about its spin-axis which is a principal axis of inertia. **Iy** is the second moment about any transverse principal axis through the CG of the bullet. The ratio **Iy/Ix** ranges from about **7** to about **14** for rifle bullets.

The coning rate $\omega_2(t)$ of a rifle bullet monotonically slows rather gradually from its launch value throughout its supersonic flight as the spin-rate of the bullet **ω(t)** slows and the ratio **R(t)** of its gyroscopic rates increases. We are not really considering subsonic flight here.

During the supersonic portion of the rifle bullet's flight in flat firing, its coning distance **D(t)** typically varies from about **4 calibers** at launch to about **10 calibers** when the bullet has slowed to **Mach 1.20** where transonic buffeting begins for many rifle bullets.

The cone apex precedes the mean CG along the mean trajectory by a distance equal to **$D(t)*COS[\alpha(t)]$**, but *the coning axis and the cone apex themselves do **not** lie directly upon the instantaneous tangent to the 3-dimensional mean trajectory, the* **+V direction**.

For a right-hand spinning bullet, the coning axis yaws horizontally rightward of the tangent to the mean trajectory by the small Horizontal Tracking Error Angle **$\varepsilon_H(t)$**. Here, the rightward yaw-of-repose attitude angle **$\beta_R(t)$** of the ***coning axis*** is causative, and the horizontal curvature of the mean trajectory is the lagging aerodynamic response. We have formulated this horizontal-direction tracking error angle **$\varepsilon_H(t)$** in Coning Theory as:

$$\varepsilon_H(t) = \beta_R(t) - \beta_T(t) \geq 0 \qquad (5)$$

where **$\beta_R(t)$** is the instantaneous yaw-of-repose angle of the bullet's coning axis, and **$\beta_T(t)$** is the instantaneous attitude angle of the tangent to the mean trajectory in the horizontal plane. Each **β-**angle is measured in radians, positive rightward in the horizontal plane from the **+x** axis of an earth-fixed **LVLH** coordinate system centered on the launch point and aligned with the launch azimuth.

The coning axis is also pitched ***vertically upward*** (nose high) in a vertical plane from the tangent to the mean trajectory at the mean CG location by the small Vertical Tracking Error Angle **$\varepsilon_V(t)$**. This vertical tracking error **$\varepsilon_V(t)$** is also dynamic in nature and is caused by the downward pitching response of the bullet's coning axis lagging in flight behind the continually downward (causative) gravitational curving of the mean trajectory in flat-firing.

This vertical tracking error angle **$\varepsilon_V(t)$** is normally quite small in flat firing, but a significantly larger version of this same vertical tracking error angle **$\varepsilon_V(t)$** has been termed the "pitch-of-repose" by McCoy in describing the higher launch-elevation-angle firing of spin-stabilized artillery projectiles; i.e., in non-flat, indirect firing.

The vertical tracking error angle **$\varepsilon_V(t)$** can be defined as:

$$\varepsilon_V(t) = \varphi bar(t) - \Delta\Phi(t) \geq 0 \qquad (6)$$

where **$\varphi bar(t)$** is the instantaneous pitch attitude (**φ**) of the bullet's *coning axis*, and **$\Delta\Phi(t)$** is the instantaneous change since launch in the Vertical Flight Path Angle (**Φ**) of the tangent to the mean trajectory. Each angle is measured at the mean CG location in radians, positive upward from the horizontal. Since **$\Delta\Phi(t)$** always exceeds **$\varphi bar(t)$** in magnitude as both angles become increasingly negative, **$\varepsilon_V(t)$** is always positive in flat-firing.

We have not yet formulated the pitch of the coning axis **$\varphi bar(t)$** in Coning Theory, but the magnitude of **$\varepsilon_V(t)$** should turn out to equal that of **$\varepsilon_H(t)$** due to their aerodynamic similarity and identical lag-time constants. The stimulus-response lag-times producing these two dynamic angular tracking errors, **$\varepsilon_V(t)$** and **$\varepsilon_H(t)$**, are each on the order of one half the period **$T_2 = 2\pi/\omega_2(t)$** of the bullet's coning motion, or **$\pi/\omega_2(t)$** seconds.

Earth-Fixed Coordinate System

We can define a rotating (non-inertial), right-handed, earth-fixed rectangular "Local Vertical-Local Horizontal (**LVLH**)" coordinate system with its origin fixed to the rotating earth at the firing point. We can utilize the precepts of classical mechanics as if we were operating in a non-rotating inertial space if we add a synthetic "Coriolis force" acting upon *any moving object* in this rotating earth-fixed coordinate space. The equally necessary and synthetic "centrifugal force" acting upon *any object* in our rotating coordinate system is already included in the vector force-field which we commonly call the "gravitational field" of the earth. With these caveats observed, we can treat **LVLH** as a "rotating inertial" coordinate system. For analytic purposes, we often "turn off" the Coriolis effect calculations in a flight simulation run when we wish to study aeroballistic effects in isolation.

Let a horizontal **x**-axis be aligned with the firing azimuth of our rifle barrel which is elevated in flat firing at no more than 100 milliradians (5.7 degrees) above the local horizontal. Muzzle elevations of 10 to 30 milliradians are typical in long-range rifle firing. We term this type of precision-aimed long-range rifle shooting "flat firing," and strive to keep the rifle bullet's airspeed supersonic (above Mach 1.2) all the way to its distant target.

Thus, the trajectory of the fired rifle bullet will not deviate far from this **x**-axis during its supersonic flight so that the magnitude of its mean velocity **V(t)** can be well approximated as:

$$\{V(t)\} \approx \{dx/dt\} \qquad (7)$$

Let us define a **y**-axis in the local horizontal plane to be positive leftward from the firing direction at a right angle from the firing azimuth and term any horizontal displacement in this **y**-direction as "drift." Thus, the rightward spin-drift of a right-hand spinning rifle bullet will produce *negative* **y**-coordinate drift values.

We can complete the right-handed earth-fixed **LVLH** coordinate system by defining a **z**-axis to be positive vertically upward and term the vertical displacement as "drop." We often specify some firing height above the **xy**-plane and some non-zero muzzle elevation angle above the horizontal **x**-axis so that a trajectory simulation run can be terminated when the bullet falls through the **xy**-plane. In any case, we calculate a positive "**DROP** from bore axis" distance by mathematically projecting that bore axis and subtracting the bullet's vertical **z**-coordinate value from that projected bore-axis height for each simulation reporting time.

Recovering the Mean Trajectory

Our available PRODAS reports give CG location data in feet (to the nearest 0.01 foot) for each millisecond of simulated flight time (**t**).

The *mean trajectory* (of the "mean CG" of the bullet) can be recovered directly by low-pass digital filtering of these **y(t)** "drift" and **z(t)** "drop" data streams to remove the confounding effects of the coning motion of the bullet's CG. Ballisticians term these small modulating effects "epicyclic swerve." The downrange distance **x(t)** is not modulated by the bullet's coning motion in flat-firing. For an adequately spin-stabilized rifle bullet, its fast-mode gyroscopic nutation rate **ω_1(t)** always exceeds at least **4*ω_2(t)**, and this faster epicyclic motion of the spin-axis direction does not move the CG of the bullet by any amount detectable in these flight simulation reports.

An ***equally-weighted running-mean*** low-pass digital filter can be employed to recover the drift and drop values defining the mean trajectory. The filter width should be **2*n+1** data samples, where the integer value **n = 1 + INT[1000*π/ω_2(t)]** for a uniform 1-millisecond sampling (or reporting) interval. Unfortunately, the filter half-width **n** must increment occasionally as the coning rate **ω_2(t)** slows gradually during the simulated flight, and the first and last **n** filtered data values are never available due to filter operator end-effects. A third-order (cubic) interpolation using **4** successive data points is ideal for finding any needed mean CG locations between the reporting times.

Vertical and Horizontal Projections of the 3-D Mean Trajectory

For any given spin-stabilized rifle bullet fired nearly horizontally in typical "flat firing," the vertical **DROP(t)** of the *mean trajectory* from the projected bore axis direction at firing and the size of the horizontal *spin-drift* displacement **SD(t)** have been found to bear a ***near constant ratio*** relationship throughout long-range ballistic flight beyond the first 150 yards or so. That is, at any later time **t** during the flight, the magnitude of the spin-drift **SD(t)** is:

$$SD(t) = ScF*DROP(t) \qquad (8)$$

where **ScF** is an essentially invariant scale factor of around **1.0** to **2.5 percent** for long-range rifle bullets. **ScF** is nearly invariant over time **t** and distance **x** during any particular long-range flight. That is, ignoring the epicyclic swerve effects, the horizontal projection of the mean trajectory looks like a scaled-down version of the vertically projected trajectory rotated 90 degrees about the axis of the bore at firing time.

This relationship was discovered during analysis of PRODAS 6-DoF simulated flight data for military M118LR Special Ball 7.62 mm ammunition, but it is believed to hold true for all spin-stabilized rifle bullets in flat firing. The numerical data show this nearly invariant ratio effect quite clearly, and it is unlikely to be accidentally true only for this one particular flat-firing trajectory. Whether any aspect of this effect extends to the high-angle firing of spin-stabilized artillery projectiles remains to be investigated.

This long-range-invariant scale factor **ScF** is about **1.2 percent** for a modern long-range, low-lift, low-drag rifle bullet fired at high muzzle velocity, and is somewhat greater than **2.3 percent** for a slower, shorter, fatter, higher-lift, higher-drag 30-caliber rifle bullet.

Implications of this Proportionality Observation

The implications of this observation that spin-drift horizontal displacement is ***directly proportional*** to drop from the bore axis at all long ranges are discussed below.

Implications for Horizontal Tangent Angle

The tangent to the horizontal projection of the bullet's *mean trajectory* in the LVLH coordinate system at any time-of-flight **t** forms the angle $\beta_T(t)$ with the **+x**-axis which is itself defined by the original launch azimuth of the bullet. Thus, the angle $\beta_T(t)$ defines the instantaneous attitude of the horizontal projection of the bullet's **+V** mean velocity vector with respect to the **+x** axis. Here we are considering only the isolated horizontal spin-drift of the mean trajectory and not any wind drift, Coriolis Effect, aeroballistic jump, or epicyclic swerve (coning motion) effects.

This horizontal tangent angle $\beta_T(t)$ plus a small dynamic horizontal tracking error angle $\varepsilon_H(t)$ equals the yaw-of-repose angle $\beta_R(t)$ at any time **t** during the flight.

Since the scale factor **ScF** tends to be invariant over time-of-flight **t** and flight distance **x** beyond the first 150 yards or so, we can form the first time-derivative of **Eq. 8** for long ranges simply as:

$$dSD/dt = ScF*dDROP/dt \qquad (9)$$

From **Eq. 7** for "flat firing" nearly horizontally along the **x**-axis, we can divide through by the non-zero magnitude of the velocity $\{V(t)\}$ of the bullet at time **t** to find the horizontal tangent angle $\beta_T(t)$ of the mean trajectory:

$$\beta_T(t) \approx dSD/dx = [dSD/dt]/[dx/dt] = ScF*[dDROP/dt]/\{V(t)\} = ScF*[\Phi(t) - \Phi(0)] \qquad (10)$$

The **DROP**-velocity function **dDROP/dt** can be obtained by filtering out the coning motion of the bullet's CG from 6-DoF simulation **DROP** data or by running a 3-DoF point-mass trajectory propagator. The instantaneous magnitude ratio of the bullet's drop-velocity **dDROP/dt** to its horizontal velocity $\{V(t)\}$ is just the total change in its vertical flight path angle $\Phi(t)$ since launch as long as the magnitude of $V(t)$ remains much greater than the magnitude of the drop velocity **dDROP/dt**, as in flat firing.

Eq. 10 also indicates that throughout the bullet's typical supersonic flight in flat firing its horizontal angular deviation from the launch azimuth $\beta_T(t)$ due to spin-drift is equal to the scale factor **ScF** times the positive-downward total vertical angular change $\Delta\Phi(t)$ in its flight path angle $\Phi(t)$ from its original launch elevation angle $\Phi(0)$,

$$\beta_T(t) \approx ScF*[\Phi(t) - \Phi(0)] = ScF*\Delta\Phi(t) \qquad (11)$$

so long as $\Delta\Phi(t) \approx TAN[\Delta\Phi(t)]$.

Implications for Wind Axes Plots

For logical consistency, the ***origin*** of the wind axes plots of the bullet's ongoing spin-axis directions in non-Eulerian pitch and yaw attitude angles should be defined, *both horizontally and vertically*, as the instantaneous **+V** direction of motion of the "mean CG" of the coning bullet; i.e., the direction of its mean velocity vector (**V**). We term this smooth path in 3-space the "mean trajectory."

For many years, the vertical change $\Delta\Phi(t)$ in the flight path angle Φ of the mean trajectory has traditionally been subtracted out of the bullet's ***pitch*** attitude values before they were shown in these wind-axes plots. Logically, the much smaller horizontal tangent angle of the mean trajectory, $\beta_T(t) = ScF*\Delta\Phi(t)$ from **Eq. 10**, should also be subtracted out before plotting the bullet's ***yaw*** attitude values.

After implementing this minor change, the yaw-of-repose angle $\beta_R(t)$ itself will no longer appear in any wind axes plots. Only the horizontal tracking error angle $\varepsilon_H(t)$ will remain in the plotted yaw data values, just as the vertical tracking error angle $\varepsilon_V(t)$ currently remains in the plotted pitch data values.

The "center" of the plotted epicyclic motion of the bullet's spin-axis in any wind conditions will then show the instantaneous pointing direction of that bullet's ***coning axis*** relative to the instantaneous tangent to its *mean trajectory*, the "**+V** direction" of the mean velocity vector.

Evaluating the Scale Factor

For any given flat firing, the scale factor **ScF** can be numerically evaluated with good accuracy by ratioing (1) the net horizontal aerodynamic "lift" force which would be attributable to its yaw-of-repose attitude angle β_R acting on the free-flying bullet at some distance downrange to (2) the vertically downward-acting constant weight **Wt** of the bullet due to the acceleration of gravity. We are temporarily ignoring the partial offset of the gravitational force **Wt** by a significant vertical component of the aerodynamic drag force acting (upward) back toward the projected axis of the bore. We can accurately evaluate this invariant scale factor **ScF** by ratioing these horizontal and vertical forces $F_H(T)/F_V(T)$ because the second time-derivatives (i.e., the horizontal and vertical accelerations) of **Eq. 8** retain this same invariant scale factor ratio **ScF** at long ranges.

The range beyond 150 yards, or so, at which this ratio of forces is evaluated is not particularly critical because the horizontal lift force curving the mean trajectory rightward in spin-drift appears to remain very nearly a *constant fraction* **ScF** *of the weight* **Wt** *of the bullet regardless of the bullet's drag function* as the Mach-speed of the bullet slows during horizontal supersonic flight. For the most accurate aeroballistic evaluation of this constant force ratio **ScF**, we select the "maximum supersonic range" occurring at flight time **T** when any subject rifle bullet is calculated to have slowed to an airspeed of **1340 feet per second**, or about Mach 1.2 in this standard sea-level ICAO atmosphere. We determine the flight time **T** to this airspeed by inspecting the airspeeds **V(t)** remaining at downrange distances as shown by any properly initialized 3-DoF point-mass trajectory propagation.

From **Eq. 10**, the tangent angle β_T turns out to be equal to $ScF*[\Phi(t)-\Phi(0)]$ and this tangent angle β_T comprises about 90 percent of the yaw-of-repose angle β_R. For brevity, we write this for any time **t** when the bullet has flown farther than its first 150 yards or so, as

$$\beta_T(t) = ScF*\Delta\Phi(t) \tag{12}$$

For purposes of this practical analysis, the net vertical-direction force **Fv(T)** acting upon free-flying bullet as a "free body" at time **T** includes the weight **Wt** of the bullet minus the upward-acting cross-bore component of the bullet's rather large drag force $F_D(T)$:

$$Fv(T) = Wt - F_D(T)*Sin[\Delta\Phi(T)]$$

$$Fv(T) = Wt - [q(T)*S]*CD_\alpha(T)*\Delta\Phi(T) \tag{13}$$

where

$CD_\alpha(T)$ **=** Coefficient of Drag for this bullet flying at an angle-of-attack given by the coning angle α at the Mach-speed corresponding to an airspeed of **1340 fps**. Bear in mind that the coning angle $\alpha(t)$ itself is actually an "unknowable" *free variable* here.

$\Delta\Phi(T) = [\Phi(T) - \Phi(0)] =$ Change in vertical-plane flight path angle Φ in radians from launch at $t = 0$ to time **T**, when the rifle bullet has slowed to an airspeed of **1340 fps**. This angular change is inherently *negative*, but we are just using its magnitude here.

The drag force $F_D(t)$ is tangent to the mean trajectory in the **-V(t)** direction at any time **t** only when firing through a ***wind-free atmosphere***. We are using only the longitudinal tangent component of the drag force here whenever surface winds are present. Any crosswind produces a separate wind-drift due to the cross-track component of this drag force $F_D(t)$ as earlier formulated by Dedion in 1859.

Then for consistency we must also include in the net horizontal force $F_H(t)$ acting on the bullet at any time **t**, the similar but scaled down, cross-bore horizontal component of the bullet's drag force

$$[F_D(t)*\beta_T(t)]_H = F_D(t)*ScF*\Delta\Phi(t) \tag{14}$$

which continually pulls the spin-drift displacement **SD(t)** back toward the original launch azimuth, opposing the horizontal lift force due to the yaw-of-repose $F_L[t, \beta_R(t)]$, and thereby keeping the horizontal and vertical projections of the mean trajectory shaped similarly.

We can express the average effective aerodynamic lift-force on the coning bullet arising from the Yaw of Repose angle $\beta_R(t)$ as if the bullet were *not* coning, but simply flying with the spin-axis always aligned with the attitude of its coning axis $[\alpha(t) = 0]$. After all, it is the attitude of that coning axis which properly defines this Yaw of Repose angle. The actual average aerodynamic horizontal angle of attack in a coordinate system moving with the free-flying bullet is just the tracking error angle $\varepsilon_H(t)$. The aerodynamic lift-force attributable to this angle of attack $\varepsilon_H(t)$ keeps increasing the rightward curvature of the mean trajectory.

In earth-fixed LVLH-coordinates, the average effective aerodynamic angle of attack can be expressed as $\varepsilon_H(t)+\beta_T(t) = \beta_R(t)$.

The small rightward aerodynamic lift force acting horizontally on the bullet is actually counteracted partially by an even smaller cross-bore component of the bullet's significant aerodynamic drag force given by $q*S*CD(t)*\beta_T(t)$.

The net rightward horizontal force $F_H(T)$ acting on the flying bullet as a free body at time **T** can be formulated as:

$$F_H(T) = F_L[T, \beta_R(T)] - F_D*\beta_T(T) \approx q(T)*S*CL_\beta(T)*\beta_R(T) - q(T)*S*CD(T)*\beta_T(T) \qquad (15)$$

This combination of lift and drag forces must remain positive throughout the flight because the spin-drift displacement always increases. Each of the three angles in the horizontal plane, $\beta_R(t)$, $\beta_T(t)$, and $\varepsilon_H(t)$, always increases monotonically with ongoing time-of-flight **t**.

Having set $ScF = F_H/F_V$ at time **T**, and using $\beta_T(T)$ from **Eq. 12**,we find that

$$ScF*[Wt - F_D(T)*\Delta\Phi(T)] = F_L[T, \beta_R(T)] - F_D(T)*\beta_T(T)$$

$$= F_L[T, \beta_R(T)] - F_D(T)*ScF*\Delta\Phi(T) \qquad (16)$$

By adding $F_D(T)*ScF*\Delta\Phi(T)$ to both sides of this equation, we have

$$ScF*Wt = F_L[T, \beta_R(T)] \qquad (17)$$

$ScF*Wt \approx$ a ***constant*** at long ranges as β_R increases and airspeed **V** decreases.

Thus, the scale factor **ScF** from **Eq. 17** can be written as

$$ScF = F_H(T)/Fv(T) = F_L[T, \beta_R(T)]/Wt \qquad (18)$$

which is how we actually evaluate **ScF** at time **T** when the bullet has slowed to **1340 fps**.

The scale factor **ScF** can be evaluated either as the ratio of the actual net horizontal and vertical free-body forces, F_H/F_V, or (more simply) as the ratio of (1) a horizontal aerodynamic lift force which would be attributable to an aerodynamic angle-of-attack equal to its yaw-of-repose attitude angle $\beta_R(t)$ to (2) the constant weight of the bullet **Wt**. We select the easier and more accurate method here for analytical calculation.

At time **T** we evaluate the scale factor **ScF** for any particular bullet's flight as

$$ScF = 0.388132*[q(T)*S]*\beta_R(T)*CL_\beta(T)/Wt \qquad (19)$$

where

0.388132 = An empirically determined constant (from PRODAS data) for all firings of “normally coning” *dynamically stable* rifle bullets through any non-zero, “reasonably constant” (non-diabolical) crosswinds. This constant is numerically necessary for several reasons, chief among them that the driving horizontal lift-force $F_L[t, \beta_R(t)]$ is actually attributable only to the horizontal tracking error attitude angle $\varepsilon_H(t)$ instead of the entire yaw-of-repose angle $\beta_R(t)$. The vertical forces acting on the bullet as a free body would also include a small upward lift force attributable to the vertical tracking error angle $\varepsilon_V(t)$. Had we considered these small horizontal and vertical lift forces in developing **Eq. 17**, the numerator in **Eq. 18** would decrease significantly while the denominator would increase slightly, hence the incorporation of the fractional constant multiplier.

$q(T) = \rho*V^2(T)/2 = \rho*[1340 \text{ fps}]^2/2 =$ Dynamic pressure in pounds per square foot

$\rho =$ Density of ambient atmosphere in slugs per cubic foot

$S = \pi*d^2/4 =$ Frontal cross-sectional area of bullet at the base of its ogive in square feet

$\beta_R(T) =$ Yaw-of-repose angle of the bullet’s coning axis at time **T** in radians

$CL_\beta(T) =$ Small-yaw coefficient of lift of the bullet at the Mach-speed corresponding to **1340 fps** airspeed, and

Wt = Weight of bullet in pounds-force, lbf.

Implications for Yaw-of-Repose Angle

If the calculated scale factor **ScF** is actually invariant during any particular long-range flight of a rifle bullet in flat firing, we can determine the yaw-of-repose $\beta_R(t)$ for any flight time **t** by solving a time-extended version of **Eq. 19** for that function:

$$\beta_R(t) = K/[V^2(t)*CL_\beta(t)] \qquad (20)$$

where the constant **K** is calculated in units of feet^2/second^2 as

$$K = 2*ScF*Wt/(0.388132*\rho*S) \qquad (21)$$

Evaluating the yaw-of-repose angle $\beta_R(t)$ from **Eq. 20** yields a function very similar to that of the (adjusted) classic approximate formulation:

$$\beta_R(t) = \pi*P*G/M \qquad (22)$$

Each formulation yields about the same ***non-zero*** value $\beta_R(0) \approx 0.130$ **milliradians** at firing time (**t = 0**). This small yaw-of-repose attitude $\beta_R(0)$ is the aerodynamic angle-of-attack which would have been required to produce the constant horizontal lift force **ScF*Wt** on the bullet at its launch time (**t = 0**). Of course, we know there is actually no such side force of this type at bullet launch.

Eq. 20 also shows that the coning bullet's yaw-of-repose angle $\beta_R(t)$ at target impact in flat firing is inversely proportional to the ***square*** of the bullet's retained velocity **V(t)** on impact with the target and is also inversely proportional to the bullet's small-yaw coefficient of lift $CL_\beta(t)$ at impact Mach-speed. The yaw-of-repose angle $\beta_R(t)$ increases continually as both $V^2(t)$ and $CL_\beta(t)$ decrease steadily in long-range flat firing.

Implications for Spin-Drift

Taken together, the implications of **Eq. 8** and **Eq. 19** determine the bullet and rifle characteristics which affect the size of the horizontal spin-drift **SD(t)** which will be seen in flat firing at a long-range target.

First, we see from **Eq. 8** that long-range spin-drift displacement **SD(t)** is always proportional to the bullet's **DROP(t)** in distance units from the projected axis of the bore at firing. This implies that modern lighter-weight "flat shooting" bullets fired at higher muzzle velocities **V(0)** and retaining more velocity farther downrange (higher ballistic coefficient, lower drag bullets) will produce much less spin-drift **SD(t)** at any given target distance compared to slower, higher-drag bullets. That is, here **SD(t)** is roughly proportional to the square of the time-of-flight **t** to the target distance.

Second, according to **Eq. 19**, the size of the scale factor **ScF**, and thence the size of the spin-drift **SD(t)**, varies directly with the "potential ballistic drag force" $q(t)*S = \rho*V^2(t)*S/2$ in pounds. The ambient atmospheric density ρ varies with shooting conditions. The rifle bullet's retained velocity **V(t)** depends upon its muzzle velocity **V(0)**, its mass **m**, and the integrated drag function CD_α of that bullet. The bullet's cross-sectional area $S = \pi*d^2/4$ varies with the square of the bullet's caliber **d.**

Third, the spin-drift **SD(t)** of the bullet is proportional to its yaw-of-repose angle $\beta_R(t)$ throughout its flight:

$$\beta_R(t) = (2\pi*g/t)\int[\omega_2(t)*V(t)]^{-1}\,dt$$

Both the coning rate $\omega_2(t)$ and the forward velocity **V(t)** of the bullet are always gradually decreasing, continually increasing $\beta_R(t)$ throughout the bullet's flight. The coning rate $\omega_2(t)$ is determined by the bullet's fixed inertial ratio **Iy/Ix** and by its remaining spin-rate $\omega(t)$ and the slowly increasing gyroscopic stability **Sg** of the flying bullet. The forward velocity **V(t)** of the flying bullet depends on its launch velocity **V(0)** and its coefficient-of-drag profile in the prevailing atmosphere.

The yaw-of-repose attitude angle $\beta_R(t)$ is *increased* for bullets having larger numerical **Iy/Ix** ratios and higher initial stability **Sg**, but $\beta_R(t)$ is *decreased* by using faster twist-rate barrels and higher muzzle velocities **V(0)** to achieve that higher gyroscopic stability **Sg**.

Fourth, the spin-drift SD(t) is directly proportional to the small-yaw coefficient of lift $CL_\beta(t)$ of the bullet. Very-low-drag (VLD) and ultra-low-drag (ULD) bullet designs usually have correspondingly reduced coefficient-of-lift functions at all supersonic airspeeds.

Fifth, and lastly, the spin-drift SD(t) of the bullet is inversely proportional to the weight Wt (or mass m) of that bullet. All else being equal, bullets made with lower average material densities, such as turned brass bullets, will weigh less and thus suffer greater spin-drift.

These five SD effects combine multiplicatively in this analysis. Some bullet and rifle design parameters recur in several of these different SD effects, and not always working in the same direction.

As modern long-range rifles and their bullets seem to be evolving toward lighter-weight, smaller-caliber, lower-drag bullets fired at higher spin-rates and at higher muzzle velocities, these related incremental variations in design parameters combine algebraically to ***reduce the spin-drift*** SD ***occurring on long-range targets***.