What is the maximum length of a spin stabilized projectile

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WHAT IS THE MAXIMUM LENGTH OF A SPIN STABILIZED PROJECTILE?

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Among others, the well known gyroscopic stability factor of a projectile depends on its geometric and physical datas as for example the moments of inertia. By calculating these moments and estimating the overturning moment by Munk's formula, we are able to determine the gyroscopic stability factor as a function of the length and the slenderness of the projectile. For simple shapes we get a closed formula. For more complicated shapes the function can be calculated with a numerical method with a computer programm.
For a given length of a projectile, we are able to determine the shape with the highest possible stability factor. For a given stability factor, the maximal length of a projectile can be calculated.

Introduction

The aerodynamic design of a projectile depends on many parameters, first of all its future use. Will it be a long range shell or a cargo shell? For a long range shell we have to choose a slender shape but for a cargo shell we decide on a body of the projectile of a big size.

In both cases, we will design the projectile as long as possible, as a long projectile gives both, a great volume and the possibility of a slender shape. Long shells still have an other advantage. The ballistic coefficient increases and the air drag consequently decreases.

A very important thing we have to consider designing such a kind of projectile is the gyroscopic stability factor, a necessary condition for its stable flight.

The gyroscopic stability factor depends only on geometrical dimensions and physical properties of the projectile and the gun, such as the moments of inertia (J_a, J_q), the diameter (d) and the gun twist angle (Δ) exept the derivative of the overturning moment coefficient c_m' (and the density of air r_l).

$$s = \frac{8}{r_l \pi} \; \frac{J_a^2 \tan^2\Delta}{J_q \, c_m' \, d^5}$$

Munk's Formula

However, in the theory of a steady, laminar flow, there exists the well known Munk's formula, that we can use to estimate the overturning moment of a body of revolution, needing only its geometrical dimensions.

$$M = \frac{r_l}{2} v^2 \sin(2a)(V - F x_s^*) \quad (2)$$

wherein means:

r_l = the air density
v = velocity
a = angle of attack
V = volume
F = cross sectional area of the end of the projectile
x_s^* = distance between the centre of gravity and the end of the projectile

From formula (2) we get easily a formula for the derivative of the overturning moment coefficient.

It is now possible to calculate the gyroscopic stability factor of a projectile by depending only on its geometrical dimensions. If the projectile is homogeneously and simply shaped, it is practicable to derive a closed formula for the stability factor.

Projectile with conical front part and cylindrical rear part

There are now the following variables we can use for the geometrical design of a projectile (see figure 1);

the total length	l
the length of the front part	h
the caliber	d

We set $n = l/d$ the length in caliber
and $k = h/l$ the front part as a fraction of the total length.

With some calculations we receive the following formulas:

Mass: $m = \frac{\pi}{4} r_p \cdot d^3 \cdot n \cdot (1 - 2k/3)$

Moments of intertia: $J_a = \frac{\pi}{32} r_p \cdot d^5 \cdot n \cdot (1 - 4k/5)$

$J_q = \frac{\pi}{960} r_p \cdot d^5 \cdot n \cdot f_1(n,k)$

Derivative: $c_m' = n \cdot f_2(k)$

with

$$f_1(n,k) = 15 - 12k + n^2 \cdot \frac{60 - 160k + 180k^2 - 96k^3 + 19k^4}{3 - 2k}$$

$$f_2(k) = \frac{18 - 24k + 7k^2}{6(3 - 2k)}$$

Then we get for the gyroscopic stability factor

$$s(n,k) = \frac{3}{10} \cdot \frac{r_p}{r_l} \cdot \tan^2\Delta \cdot \frac{(5 - 4k)^2}{f_1(n,k)\ f_2(k)} \qquad (3)$$

(r_p: density of projectile)

Discussing this function, we remark that for a given length n there exist a k for witch the stability becomes maximal (see figure 2). It is also possible to design a shape with maximal stability.

Comparison with wind-tunnel tests

It was an interesting question, if this feature was a result of the use of Munk's formula or if a real projectile with measured derivative of the overturning moment coefficient also had a maximum of stability.

To answer this question we made wind-tunnel tests with 3 projectiles with a length of 4.5 caliber and a ratio k of 0, 0.5, 0.66 and 1.0. We got the following results:

For supersonic and subsonic velocities the characteristic of the measured c_m' and of the Munk-calculated c_m' was the same (see figure 3). Hence, the stability factor, based on the measured c_m' has also a maximum (see figure 4).

But we saw that Munk's formula gives a too high value for the derivative of the overturning moment coefficient and therefor a too low stability factor.

For transsonic velocities, the characteristic of the measured c_m' was different of the calculated one, but one could expect this result.

What's the maximum length of a spinning projectile?

To examine the influence of the shape of the projectile, we calculated the stability factor as a function of the geometrical dimensions for two types of projectiles with a computer program.

"shell-type": steel case, filled with explosif
"bullet-type": steel jacket, lead core

For both types we had planned a boat-tailed rear part and several different shaped front parts as

- an ogive
- a form with minimum forebody drag
- a cone

We remarked that for all kind of projectiles there exists a ratio of the front part length to the total length so that the stability becomes maximal. The position of that maximum depends on the shape but not on the total length of the projectile (see figure 6 and 7).

Using the shape with maximal stability, we calculated the relation between the angle of twist, the total length and the gyroscopic stability factor. The results of these calculations are shown in the figures 8 and 9 whereby the values of Munk's formula had been proportionned to the wind-tunnel measurements.

Conclusion

If we have to design a projectile for a given angle of twist that we want to make as long as possible, we are able to determine a shape with the highest possible stability.

If we are also free in the choice of the twist, and that already at the moment of design, we can optimize the shape of the projectile and at the same time also the twist.

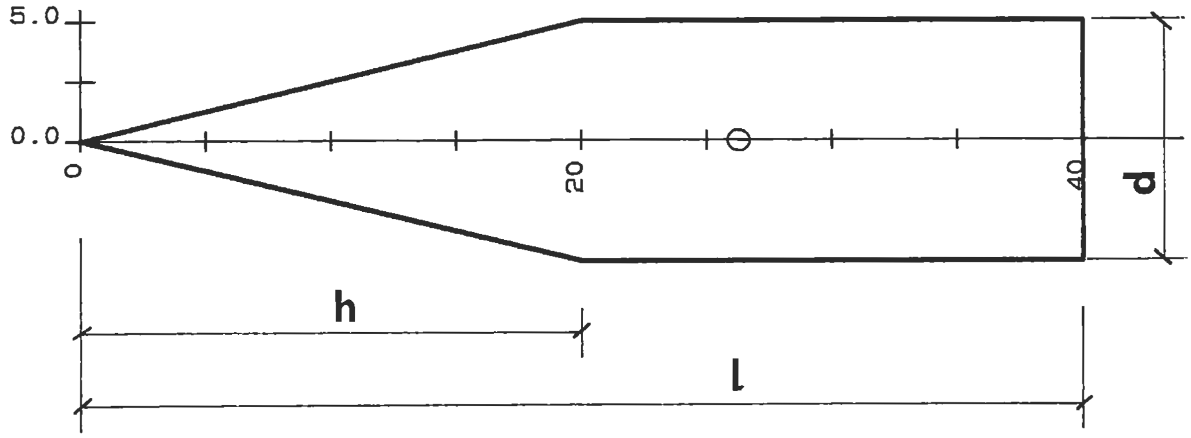


FIGURE 1

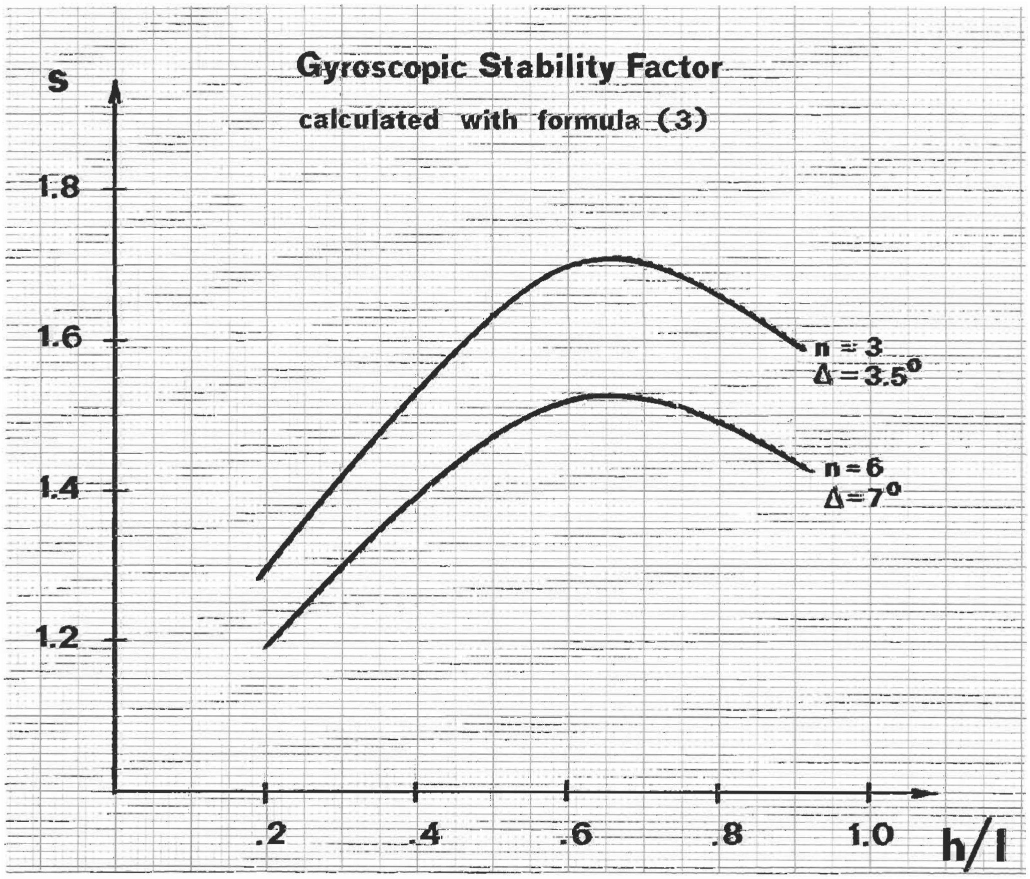


FIGURE 2

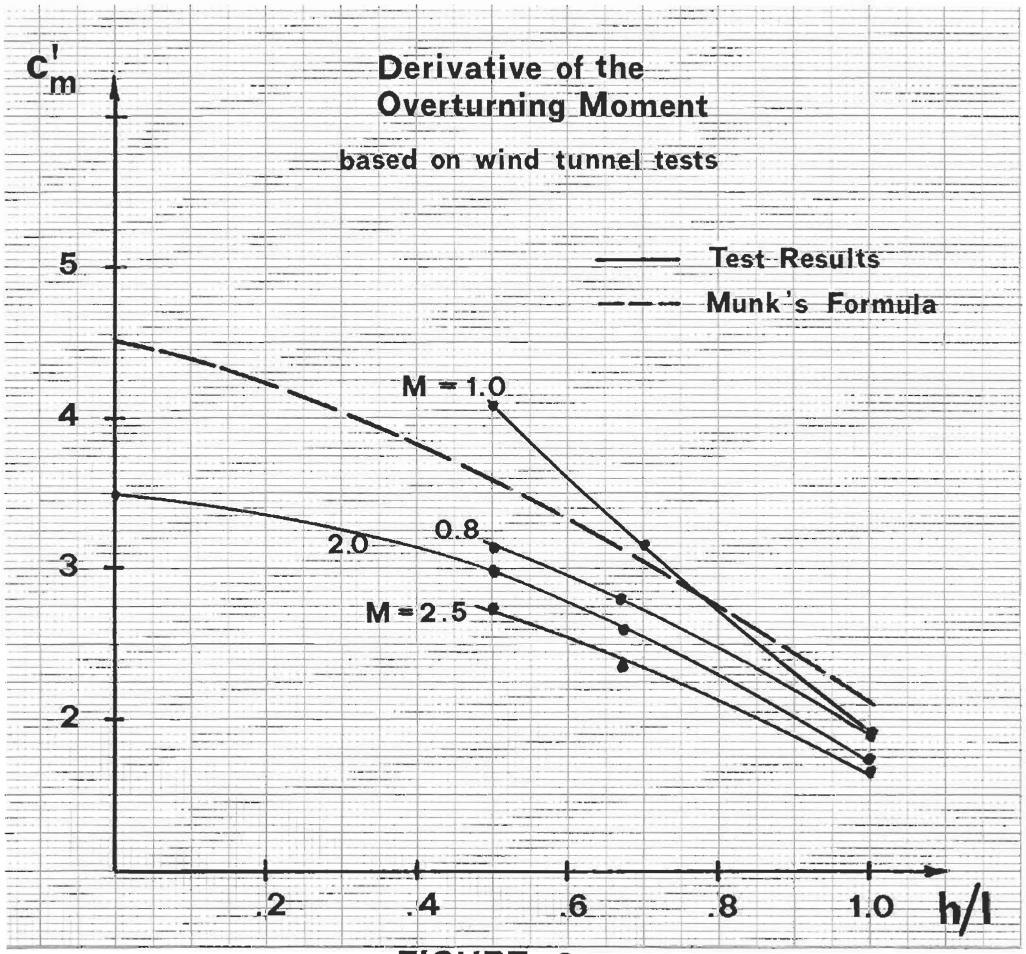


FIGURE 3

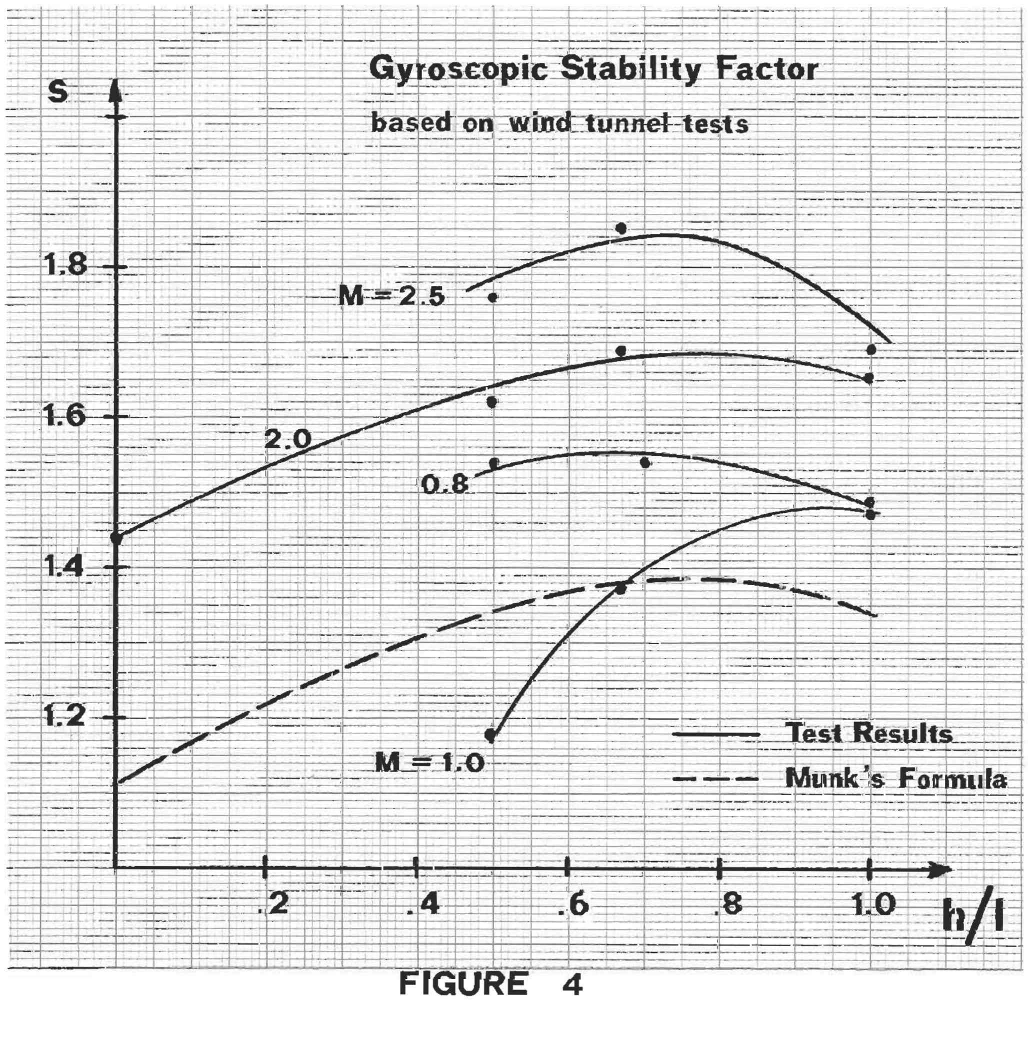
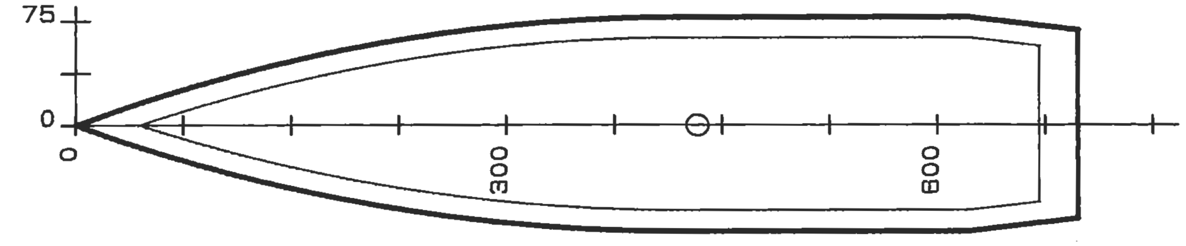
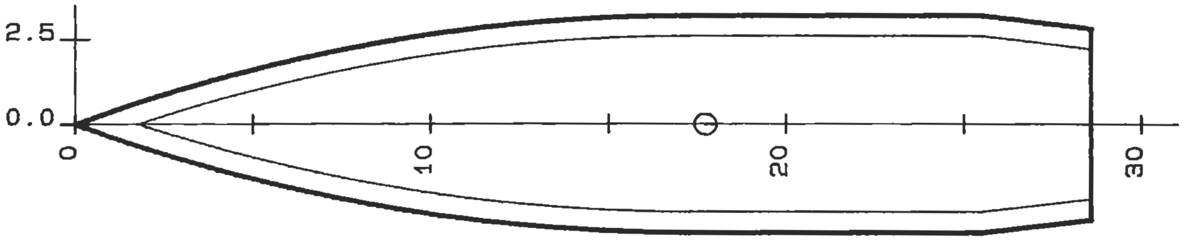


FIGURE 4



Shell Type



Bullet Type

FIGURE 5

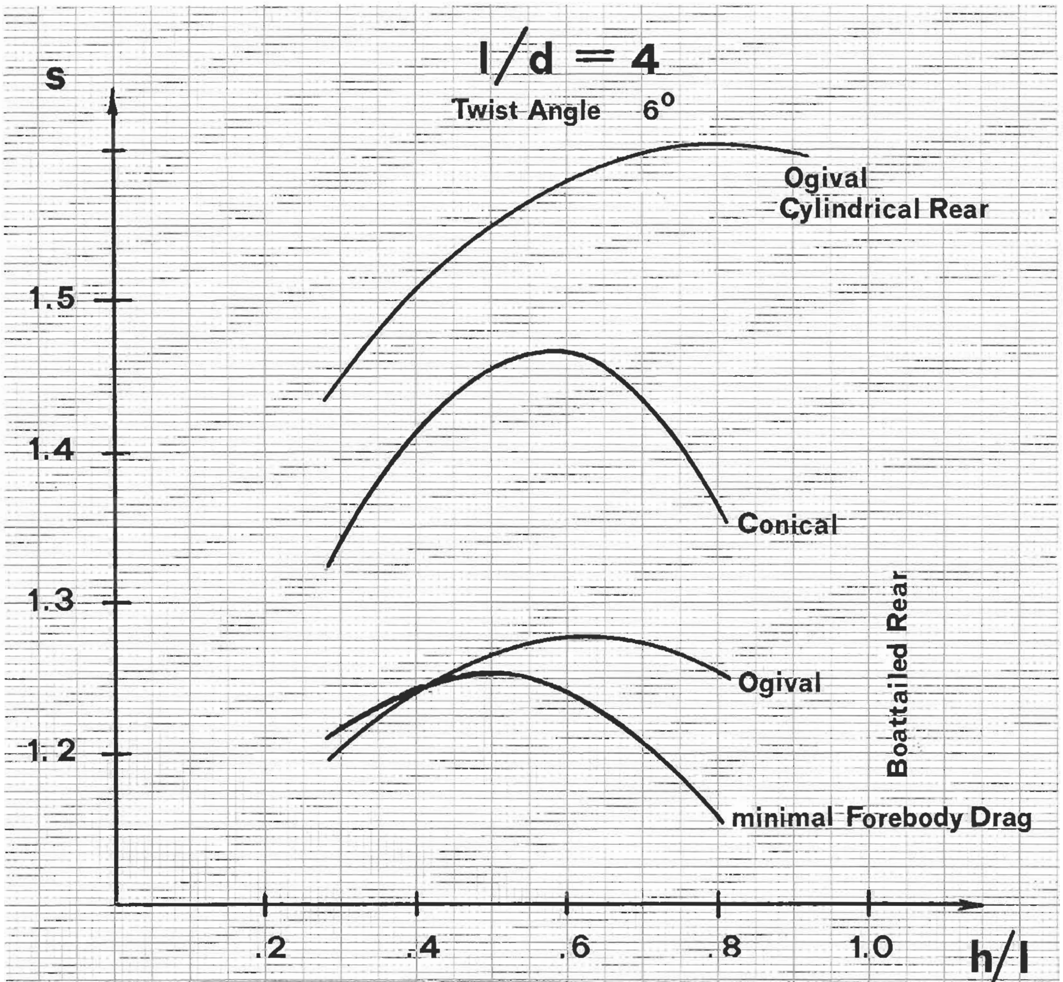


FIGURE 6

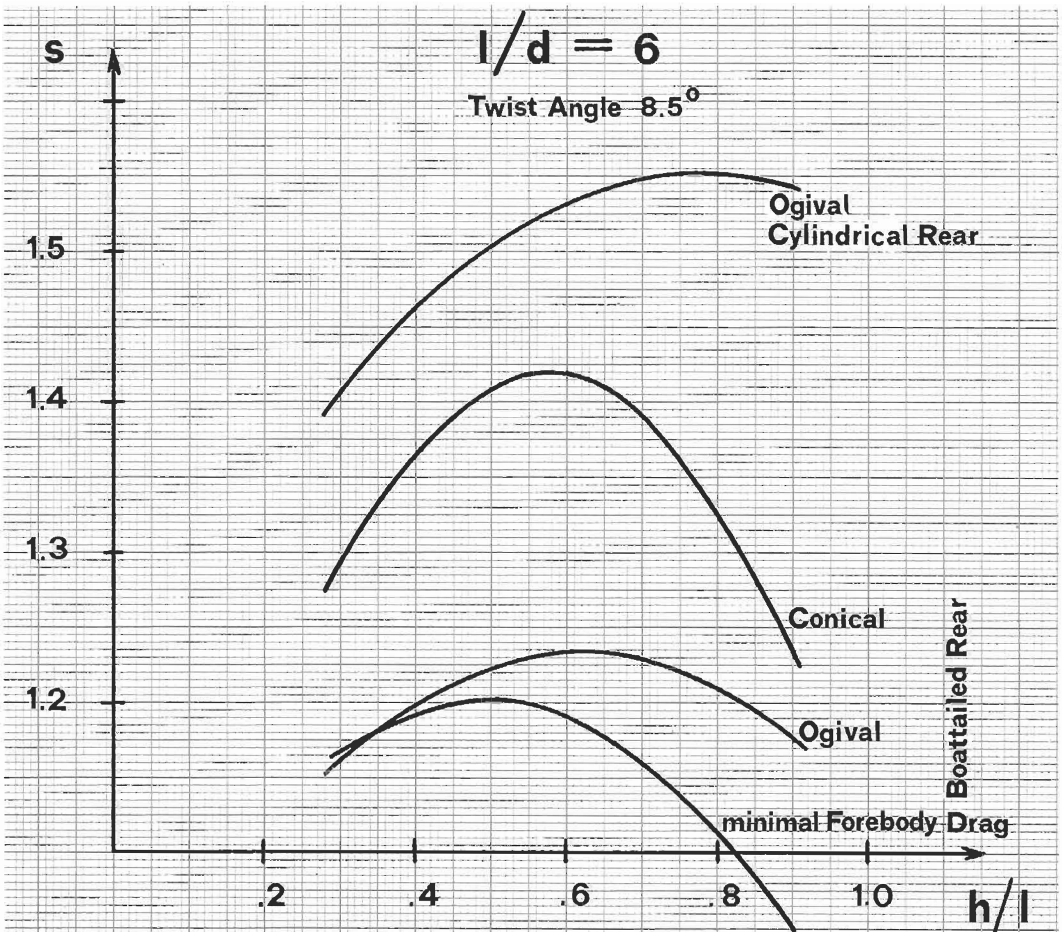


FIGURE 7

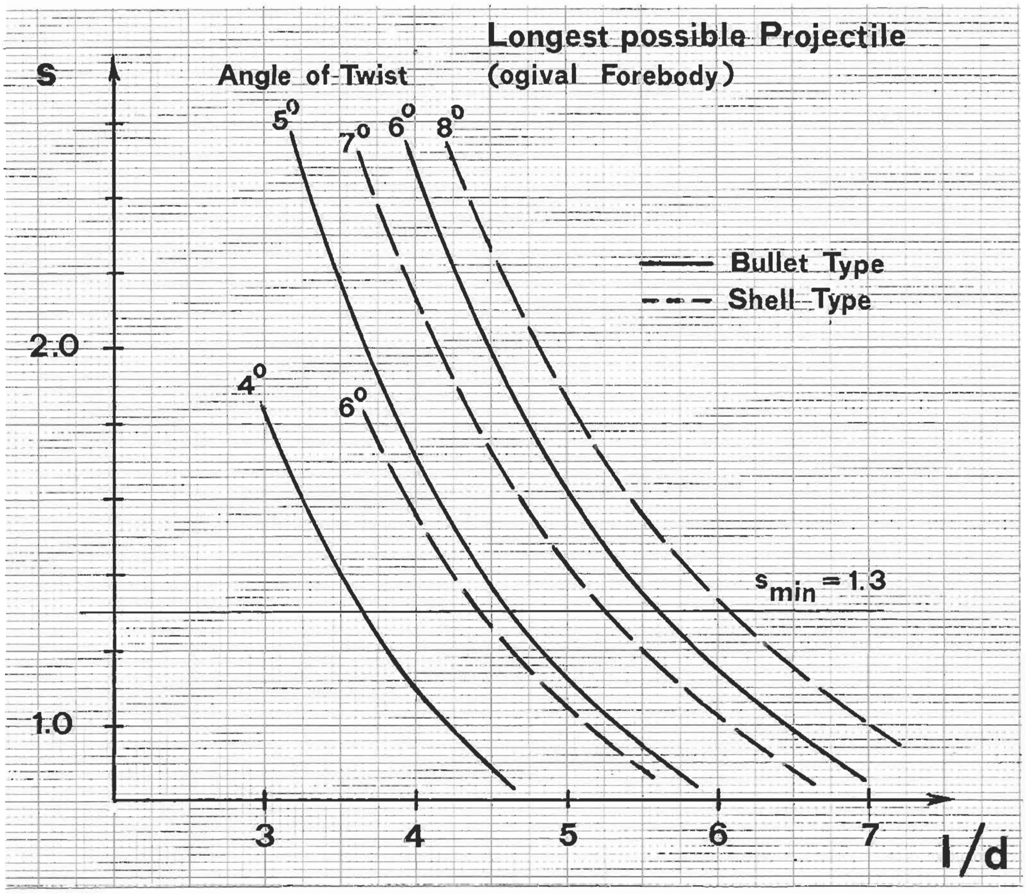


FIGURE 8

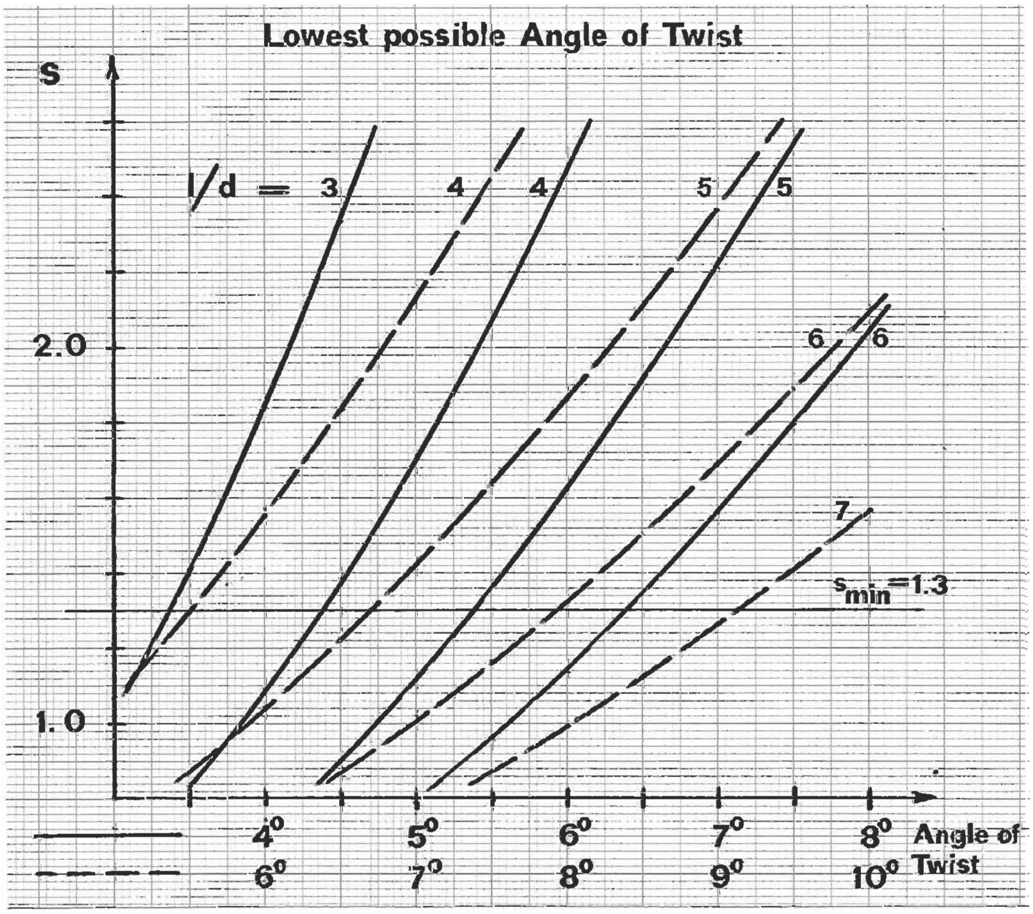


FIGURE 9